MOTION IN A PLANE

FACT/DEFINITION TYPE QUESTIONS

1. A scalar quantity is one that
(a) can never take negative values
(b) has magnitude as well as direction
(c) does not vary from one point to another in space
(d) has the same value for observers with different orientations of axes.

2. Which of the following conditions are sufficient and essential for a quantity to be a vector?
(a) Magnitude and direction
(b) Magnitude and addition, subtraction, multiplication by ordinary rules of algebra
(c) Magnitude, direction, and addition, subtraction multiplication and division by vector laws
(d) Magnitude, direction and combination of vectors by ordinary rules of algebra

3. If θ is the angle between two vectors, then the resultant vector is maximum, when value of θ is
(a) 0° (b) 90°
(c) 180° (d) Same in all cases.

4. How many minimum number of vectors in different planes can be added to give zero resultant?
(a) 2 (b) 3
(c) 4 (d) 5

5. The unit vectors along the three co-ordinate axes are related as
(a) $\hat{i} > \hat{j} > \hat{k} > 1$ (b) $\hat{i} = \hat{j} = \hat{k} = 0$
(c) $\hat{i} = -\hat{j} = \hat{k} = 1$ (d) $\hat{i} = \hat{j} = \hat{k} = 1$

6. The angle between the direction of $\hat{i}$ and $(\hat{i} + \hat{j})$ is
(a) 90° (b) 0°
(c) 45° (d) 180°

7. Consider the quantities pressure, power, energy, impulse, gravitational potential, electric charge, temperature and area. Out of these, the vector quantities are
(a) impulse, pressure and area
(b) impulse and area
(c) area and gravitational potential
(d) impulse and pressure

8. Angular momentum is
(a) a scalar (b) a polar vector
(c) an axial vector (d) None of these

9. The resultant of $\vec{A} \times 0$ will be equal to
(a) zero (b) A
(c) zero vector (d) unit vector

10. In a clockwise system
(a) $\hat{j} \times \hat{k} = \hat{i}$ (b) $\hat{i} \times \hat{k} = 0$
(c) $\hat{j} \times \hat{j} = 1$ (d) $\hat{k} \times \hat{i} = 1$

11. The component of a vector r along x-axis will have maximum value if
(a) r is along positive y-axis
(b) r is along positive x-axis
(c) r makes an angle of 45° with the x-axis
(d) r is along negative y-axis

12. It is found that $|A + B| = |A|$. This necessarily implies,
(a) B = 0
(b) A and B are antiparallel
(c) A and B are perpendicular
(d) $A.B \leq 0$

13. The shape of trajectory of the motion of an object is determined by
(a) acceleration (b) initial position
(c) initial velocity (d) All of these

14. The position vector a of particle is

$$\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$$

The velocity of the particle is
(a) directed towards the origin
(b) directed away from the origin
(c) parallel to the position vector
(d) perpendicular to the position vector

15. If t_m is the time taken by a projectile to achieve the maximum height, then the total time of flight T_f related to t_m as
(a) $t_m = 2\,T_f$ (b) $T_f = t_m$
(c) $T_f = 2t_m$ (d) None of these

16. If u is the initial velocity of a projectile and v is the velocity at any instant, then the maximum horizontal range R_m is equal to
(a) $R_m = \dfrac{u^2 \sin 2\theta}{g}$ (b) $R_m = \dfrac{v^2}{g}$
(c) $R_m = \dfrac{v^2 \sin 2\theta}{g}$ (d) $R_m = \dfrac{u^2}{g}$

17. Which of the following is an essential condition for horizontal component of projectile to remain constant?
(a) Acceleration due to gravity should be exactly constant
(b) Angle of projection should be 45°
(c) There should be no air-resistance
(d) All of these

18. In the projectile motion, if air resistance is ignored, the horizontal motion is at
(a) constant acceleration (b) constant velocity
(b) variable acceleration (d) constant retardation

19. A moves with 65 km/h while B is coming back of A with 80 km/h. The relative velocity of B with respect to A is
(a) 80 km/h (b) 60 km/h
(c) 15 km/h (d) 145 km/h

20. A bullet is dropped from the same height when another bullet is fired horizontally. They will hit the ground
(a) one after the other
(b) simultaneously
(c) depends on the observer
(d) None of these

21. What determines the nature of the path followed by a particle?
(a) Velocity (b) Speed
(c) Acceleration (d) None of these

22. The time of flight of a projectile on an upward inclined plane depends upon
(a) angle of inclination of the plane
(b) angle of projection
(c) the value of acceleration due to gravity
(d) all of the above.

23. At the highest point on the trajectory of a projectile, its
(a) potential energy is minimum
(b) kinetic energy is maximum
(c) total energy is maximum
(d) kinetic energy is minimum.

24. In a projectile motion, velocity at maximum height is
(a) $\frac{u\cos\theta}{2}$ (b) $u\cos\theta$
(c) $\frac{u\sin\theta}{2}$ (d) None of these

25. The angle of projection, for which the horizontal range and the maximum height of a projectile are equal, is:
(a) 45° (b) $\theta = \tan^{-1}4$
(c) $\theta = \tan^{-1}(0.25)$ (d) none of these.

26. For an object thrown at 45° to horizontal, the maximum height (H) and horizontal range (R) are related as
(a) $R = 16\text{ H}$ (b) $R = 8\text{ H}$
(c) $R = 4\text{ H}$ (d) $R = 2\text{H}$

27. The vertical component of velocity of a projectile at its maximum height (u - velocity of projection, θ-angle of projection) is
(a) $u\sin\theta$ (b) $u\cos\theta$
(c) $\frac{u}{\sin\theta}$ (d) 0

28. For projectile motion, we will assume that the air resistance has ...X... effect on the motion of the projectile. Here, X refers to
(a) sufficient (b) insufficient
(c) negligible (d) proper

29. For angle ...X..., the projectile has maximum range and it is equal to ...X.... Here, X and Y refer to
(a) $\frac{\pi}{4}$ and $\frac{v_0^2}{2g}$ (b) $\frac{\pi}{2}$ and $\frac{v_0}{g}$
(c) $\frac{\pi}{4}$ and $\frac{v_0^2}{g}$ (d) $\frac{\pi}{2}$ and $\frac{v_0^2}{g}$

30. At the top of the trajectory of a projectile, the acceleration is
(a) maximum (b) minimum
(c) zero (d) g

31. Centripetal acceleration is
(a) a constant vector
(b) a constant scalar
(c) a magnitude changing vector
(d) not a constant vector

32. The force required to keep a body in uniform circular motion is
(a) centripetal force (b) centrifugal force
(c) resistance (d) None of these

33. In a vertical circle of radius r at what point in the path a particle has tension equal to zero if it is just able to complete the vertical circle?
(a) Highest point
(b) Lowest point
(c) Any point
(d) At a point horizontally from the centre of circle of radius r

34. Two stones are moving with same angular speeds in the radii of circular paths 1 m and 2 m. The ratio of their linear speed is ...X.... Here, X refers to
(a) 2 (b) 1/2
(c) 1/3 (d) 3

35. The direction of the angular velocity vector is along
(a) the tangent to the circular path
(b) the inward radius
(c) the outward radius
(d) the axis of rotation

36. If a_r and a_t represent radial and tangential accelerations, the motion of particle will be uniformly circular, if
(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ but $a_t \neq 0$
(c) $a_r \neq 0$ and $a_t = 0$ (d) $a_r \neq 0$ and $a_t \neq 0$

37. In uniform circular motion
(a) both velocity and acceleration are constant
(b) acceleration and speed are constant but velocity changes
(c) both acceleration and velocity change
(d) both acceleration and speed are constant

38. When a body moves with a constant speed along a circle
(a) no work is done on it
(b) no acceleration is produced in the body
(c) no force acts on the body
(d) its velocity remains constant

39. A body is travelling in a circle at a constant speed. It
(a) has a constant velocity
(b) is not accelerated
(c) has an inward radial acceleration
(d) has an outward radial acceleration

40. A body is moving with a constant speed v in a circle of radius r. Its angular acceleration is
(a) vr (b) v/r
(c) zero (d) vr^2

41. A stone of mass m is tied to a string of length ℓ and rotated in a circle with a constant speed v, if the string is released the stone flies
(a) radially outward
(b) radially inward
(c) tangentially outward
(d) with an acceleration mv^2/ℓ

42. If a particle moves in a circle describing equal angles in equal interval of time, its velocity vector
(a) remains constant
(b) changes in magnitude
(c) changes in direction
(d) changes both in magnitude and direction

43. The circular motion of a particle with constant speed is
(a) periodic but not simple harmonic
(b) simple harmonic but not periodic
(c) periodic and simple harmonic
(d) neither periodic nor simple harmonic

44. In uniform circular motion, the velocity vector and acceleration vector are
(a) perpendicular to each other
(b) same direction
(c) opposite direction
(d) not related to each other

45. A body of mass m moves in a circular path with uniform angular velocity. The motion of the body has constant
(a) acceleration (b) velocity
(c) momentum (d) kinetic energy

STATEMENT TYPE QUESTIONS

46. Consider the following statements and select the correct statements from the following.
I. Addition and subtraction of scalars make sense only for quantities with same units
II. Multiplication and division of scalars with different units is possible
III. Addition, subtraction, multiplication and division of scalars with same unit is possible
(a) I and II (b) II and III
(c) I and III (d) I, II and III

47. Which of the following statements is/are incorrect?
I. A scalar quantity is the one that is conserved in a process.
II. A scalar quantity is the one that can never take negative values.
III. A scalar quantity has the same value for observers with different orientations of the axes.
(a) I and III (b) II only
(c) II and III (d) I and II

48. Which of the following is/are correct ?
I. $\vec{A} \times \vec{B} = -\vec{B} \times A$
II. $\vec{A} \times \vec{B} \neq \vec{B} \times A$
III. $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + \vec{C}$
(a) I only (b) II and III
(b) I and III (d) I and II

49. Three vectors **A, B** and **C** add up to zero. Select the correct statements.
I $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ is not zero unless **B, C** are parallel
II If **A,B,C** define a plane, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ is in that plane
III $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = |\mathbf{A}||\mathbf{B}||\mathbf{C}| \rightarrow \mathbf{C}^2 = \mathbf{A}^2 + \mathbf{B}^2$
(a) I and II (b) II and III
(c) I and III (d) I, II and III

50. In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following statements is/are incorrect?
I. The average velocity is not zero at any time.
II. Average acceleration must always vanish.
III. Equal path lengths are traversed in equal intervals.
(a) I and II (b) II only
(c) III only (d) II and III

51. Choose the correct statement(s) from the following.
I. If speed of a body in a curved path is constant it has zero acceleration
II. When a body moves on a curved path with a constant speed, it has acceleration perpendicular to the direction of motion
(a) I only (b) II only
(c) I and II (d) None of these

52. Select the correct statements about the football thrown in a parabolic path.
I. At the highest point the vertical component of velocity is zero
II. At the highest point, the velocity of the football, acts horizontally
III. At the highest point, the acceleration of the ball acts vertically downwards
(a) I and II (b) II and III
(c) I and III (d) I, II and III

53. Select the incorrect statement(s) from the following.
I. In projectile motion, the range depends on the mass. It is greater for heavier object
II. In projectile motion, the range is independent of the angle of projection.
(a) I only (b) II only
(c) I and II (d) None of these

54. A ball is thrown upwards and it returns to ground describing a parabolic path. Which of the following has the same value at the time of throw and the time of return?

I. Kinetic energy of the ball

II. Speed of the ball

III. Vertical component of velocity

(a) I and II (b) II and III

(c) III only (d) I, II and III

55. For a particle performing uniform circular motion, select the correct statement(s) from the following.

I. Magnitude of particle velocity (speed) remains constant.

II. Particle velocity remains directed perpendicular to radius vector.

III. Angular momentum is constant.

(a) I only (b) II and III

(c) III only (d) I and II

56. Which of the following statements are correct ?

I. Centripetal acceleration is always directed towards the centre.

II. Magnitude of the centripetal acceleration is $\frac{v^2}{R}$.

III. Direction of centripetal acceleration changes pointing always towards the centre.

(a) I and II (b) II and III

(c) I and III (d) I, II and III

MATCHING TYPE QUESTIONS

57. Vector $\vec{A}$ has components $A_x = 2$, $A_y = 3$ and vector $\vec{B}$ has components $B_x = 4$, $B_y = 5$, then match the columns :

Column I	Column II
(A) The components of vector sum $(\vec{A}+\vec{B})$	(1) 8
(B) The magnitude of $\vec{A}+\vec{B}$	(2) – 2
(C) The componet of vector difference $\vec{A}-\vec{B}$	(3) $2\sqrt{2}$
(D) The magnitude of $(\vec{A}-\vec{B})$	(4) 10

(a) (A)→(1); (B)→(4); C→(2); (D)→(3)

(b) (A)→(2); (B)→(4); C→(3); (D)→(1)

(c) (A)→(3); (B)→(2); C→(4); (D)→(1)

(d) (A)→(2); (B)→(4); C→(1); (D)→(3)

58. Given two vectors $\vec{A} = 3\hat{i}+4\hat{j}$ and $\vec{B} = \hat{i}-2\hat{j}$. Then match the following columns :

Column I	Column II
(A) Magnitude of vector $\vec{A}$ or $\vec{B}$	(1) 5
(B) Unit vector of $\vec{A}$	(2) $(0.6\hat{i}+0.8\hat{j})$
(C) The magnitude of $\vec{A}+\vec{B}$	(3) $(2\hat{i}+6\hat{j})$
(D) The difference of vector, $\vec{A}-\vec{B}$	(4) $\sqrt{20}$

(a) (A)→(4); (B)→(1); C→(2); (D)→(3)

(b) (A)→(1); (B)→(2); C→(4); (D)→(3)

(c) (A)→(3); (B)→(2); C→(4); (D)→(1)

(d) (A)→(2); (B)→(4); C→(1); (D)→(3)

59. Given two vectors; $\vec{A} = \hat{i}+\hat{j}$ and $\vec{B} = \hat{i}-\hat{j}$. Then match the following columns :

Column - I	Column - II
(A) $(\vec{A}+\vec{B})/2$	(1) $\hat{i}$
(B) $(\vec{A}-\vec{B})/2$	(2) $\hat{j}$
(C) $(\vec{A}\cdot\vec{B})/2$	(3) $-\hat{k}$
(D) $(\vec{A}\times\vec{B})/2$	(4) 0

(a) (A)→(4); (B)→(1); C→(2); (D)→(2)

(b) (A)→(2); (B)→(4); C→(3); (D)→(1)

(c) (A)→(3); (B)→(2); C→(4); (D)→(1)

(d) (A)→(1); (B)→(2); C→(4); (D)→(3)

60. The velocity $\vec{v}$ of a particle moving in the xy – plane is given by $\vec{v} = (6t-4t^2)\hat{i}+8\hat{j}$, with $\vec{v}$ in m/s and t(>0) in second.

Match the following columns :

Column - I	Column - II
(A) Acceleration magnitude is 10 m/s^2 at a time	(1) ¾ s
(B) Acceleration zero at time	(2) never
(C) velocity zero at time	(3) 1 s
(D) The speed 10 m/s at a time	(4) 2 s

(a) (A)→(4); (B)→(1); C→(2); (D)→(3)

(b) (A)→(2); (B)→(4); C→(3); (D)→(1)

(c) (A)→(3); (B)→(2); C→(4); (D)→(1)

(d) (A)→(2); (B)→(4); C→(1); (D)→(3)

61. The equation of trajectory of a particle projected from the surface of the planet is given by the equation $y = x - x^2$. (suppose, g = 2 m/s^2)

Column - I	Column - II (magnitude only)
(A) angle of projection, tan q	(1) ¼
(B) time of flight, T	(2) 1
(C) maximum height attained, H	(3) 2
(D) horizontal range, R	(4) 4

(a) (A)→(4); (B)→(1); C→(2); (D)→(2)

(b) (A)→(2); (B)→(3); C→(1); (D)→(4)

(c) (A)→(3); (B)→(2); C→(4); (D)→(1)

(d) (A)→(2); (B)→(4); C→(1); (D)→(3)

62. A particle is projected with some angle from the surface of the planet. The motion of the particle is described by the equation; $x = t$, $y = t - t^2$. Then match the following columns:

Column - I (quantity)	**Column - II** (magnitude only)
(A) velocity of projection	(1) 1
(B) acceleration	(2) $\sqrt{2}$
(C) time of flight	(3) 2
(D) maximum height attained	(4) $\frac{1}{4}$

(a) (A)→(4); (B)→(1); C→(2); (D)→(2)
(b) (A)→(2); (B)→(3); C→(1); (D)→(2)
(c) (A)→(2); (B)→(3); C→(1); (D)→(4)
(d) (A)→(3); (B)→(4); C→(3); (D)→(2)

63. A ball is thrown at an angle 75° with the horizontal at a speed of 20 m/s towards a high wall at a distance d. If the ball strikes the wall, its horizontal velocity component reverses the direction without change in magnitude and the vertical velocity component remains same. Ball stops after hitting the ground. Match the statement of column I with the distance of the wall from the point of throw in column II .

Column I	**Column II**
(A) Ball strikes the wall directly	(1) 8 m
(B) Ball strikes the ground at x = 12 m from the wall	(2) 10 m
(C) Ball strikes the ground at x = 10m from the wall	(3) 0 m
(D) Ball strikes the ground at x = 5 m from the wall	(4) 25 m

(a) (A)→(1,2); (B)→(1); C→(2); (D)→(4)
(b) (A)→(2); (B)→(3); C→(1); (D)→(2)
(c) (A)→(2); (B)→(3); C→(1); (D)→(4)
(d) (A)→(3); (B)→(4); C→(3); (D)→(2)

DIAGRAM TYPE QUESTIONS

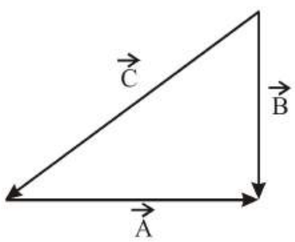
64. For the figure, which of the following is correct?

(a) $\vec{A} + \vec{B} = \vec{C}$

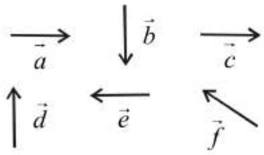
(b) $\vec{B} + \vec{C} = \vec{A}$

(c) $\vec{C} + \vec{A} = \vec{B}$

(d) $\vec{A} + \vec{B} + \vec{C} = 0$



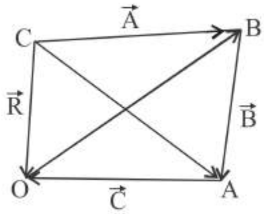
65. Six vectors, $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true?



(a) $\vec{b} + \vec{c} = \vec{f}$ (b) $\vec{d} + \vec{c} = \vec{f}$

(c) $\vec{d} + \vec{e} = \vec{f}$ (d) $\vec{b} + \vec{e} = \vec{f}$

66. Which law is governed by the given figure ?

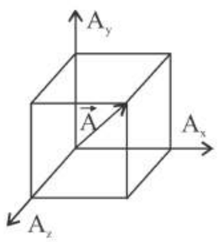


(a) Associative law of vector addition
(b) Commutative law of vector addition
(c) Associative law of vector multiplication
(d) Commutative law of vector multiplication

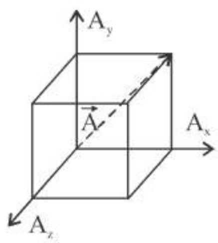
67. Which of the following figures represents

$\vec{A} = A_X\hat{i} + A_y\hat{j} + A_z\hat{k}$?

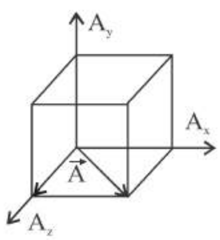
(a)



(b)

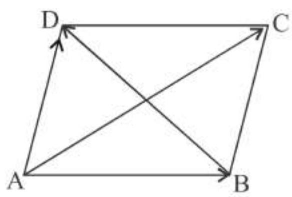


(c)



(d) All of these

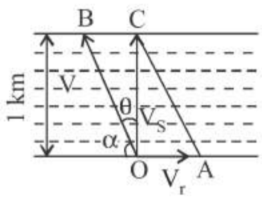
68. Which of the following holds true for the given figure?

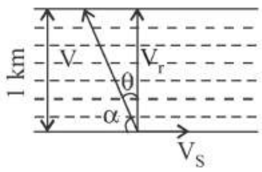


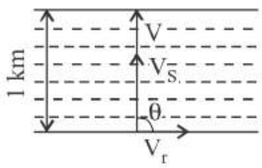
(a) $\overline{AC} + \overline{BD} = 2\overline{BC}$ (b) $\overline{AB} + \overline{BC} = 2\overline{CD}$

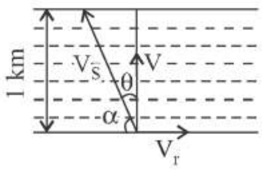
(c) $\overline{AC} - \overline{AB} = 2\overline{BD}$ (d) All of these

69. A swimmer wants to cross a river straight. He swim at 5 km/hr in still water. A river 1 km wide flows at the rate of 3 km/hr. Which of the following figure shows the correct direction for the swimmer along which he should strike? ($V_s \rightarrow$ velocity of swimmer, $V_r \rightarrow$ velocity of river, $V \rightarrow$ resultant velocity)

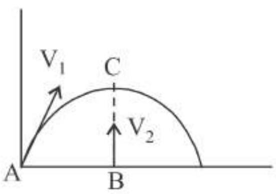
(a)


(b)


(c)


(d)


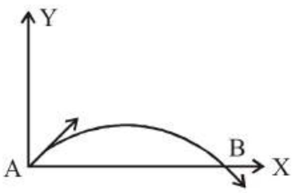
70. If V_1 is velocity of a body projected from the point A and V_2 is the velocity of a body projected from point B which is vertically below the highest point C. if both the bodies collide, then



(a) $V_1 = \frac{1}{2}V_2$ (b) $V_2 = \frac{1}{2}V_1$

(c) $V_1 = V_2$ (d) Two bodies can't collide.

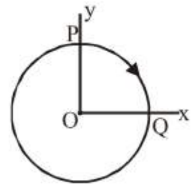
71. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j})$ m/s its velocity (in m/s) at point B is



(a) $-2\hat{i} + 3\hat{j}$ (b) $2\hat{i} - 3\hat{j}$

(c) $2\hat{i} + 3\hat{j}$ (d) $-2\hat{i} - 3\hat{j}$

72. A particle moves in a circle of radius 4 cm clockwise at constant speed 2 cm/s. If $\hat{x}$ and $\hat{y}$ are unit acceleration vectors along X and Y-axis respectively (in cm/s^2), the acceleration of the particle at the instant half way between P and Q is given by



(a) $-4(\hat{x} + \hat{y})$

(b) $4(\hat{x} + \hat{y})$

(c) $-(\hat{x} + \hat{y})/\sqrt{2}$

(d) $(\hat{x} - \hat{y})/4$

ASSERTION- REASON TYPE QUESTIONS

Directions : Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

73. Assertion: A physical quantity cannot be called as a vector if its magnitude is zero.

Reason: A vector has both magnitude and direction.

74. Assertion : The scalar product of two vectors can be zero.

Reason : If two vectors are perpendicular to each other, their scalar product will be zero.

75. Assertion : Minimum number of non-equal vectors in a plane required to give zero resultant is three.

Reason : If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then they must lie in one plane.

76. Assertion : If $\vec{A}.\vec{B} = \vec{B}.\vec{C}$, then $\vec{A}$ may not always be equal to $\vec{C}$.

Reason : The dot product of two vectors involves cosine of the angle between the two vectors.

77. Assertion : $\vec{\tau} = \vec{r} \times \vec{F}$ and $\vec{\tau} \neq \vec{F} \times \vec{r}$

Reason : Cross product of vectors is commutative.

78. Assertion : If dot product and cross product of $\vec{A}$ and $\vec{B}$ are zero, it implies that one of the vector $\vec{A}$ and $\vec{B}$ must be a null vector

Reason : Null vector is a vector with zero magnitude.

79. Assertion : The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on one bank may reach opposite bank simultaneously moving along different paths.

Reason : For boats to cross the river in same time. The component of their velocity relative to river in direction normal to flow should be same.

80. **Assertion :** Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.

Reason : The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.

81. **Assertion :** If a body of mass m is projected upwards with a speed V making an angle θ with the vertical, than the change in the momentum of the body along X–axis is zero.
Reason : Mass of the body remains constant along X–axis

82. **Assertion :** The horizontal range is same when the angle of projection is greater than 45° by certain value and less than 45° by the same value.
Reason: If $\theta = 45° + \alpha$, then

$$R_1 = \frac{u^2 \sin 2(45° + \alpha)}{g} = \frac{u^2 \cos 2\alpha}{g}$$

If $\theta = 45° - \alpha$, then $R_2 = \frac{u^2 \sin 2(45° - \alpha)}{g} = \frac{u^2 \cos 2\alpha}{g}$

83. **Assertion :** If there were no gravitational force, the path of the projected body always be a straight line.

Reason : Gravitational force makes the path of projected body always parabolic.

84. **Assertion:** The maximum possible height attained by the projected body is $\frac{u^2}{2g}$, where u is the velocity of projection.

Reason : To attain the maximum height, body is thrown vertically upwards.

85. **Assertion :** When the range of projectile is maximum, the time of flight is the largest.

Reason : Range is maximum when angle of projection is 45^0.

86. **Assertion :** A shell fired from a gun is moving along the parabolic path. If it explodes at the top of the trajectory, then no part of the shell can fly vertically.

Reason : The vertical momentum of the shell at the top of the trajectory is zero.

87. **Assertion :** A body is thrown with a velocity u inclined to the horizontal at some angle. It moves along a parabolic path and falls to the ground. Linear momentum of the body, during its motion, will remain conserve.

Reason : Throughout the motion of the body, a constant force acts on it.

88. **Assertion :** Two projectiles having same range must have the same time of flight.

Reason : Horizontal component of velocity is constant in projectile motion under gravity.

89. **Assertion :** The maximum horizontal range of projectile is proportional to square of velocity.
Reason : The maximum horizontal range of projectile is equal to maximum height attained by projectile.

90. **Assertion :** The trajectory of projectile is quadratic in *y* and linear in *x*.
Reason : *y* component of trajectory is independent of *x*-component.

91. **Assertion :** When range of a projectile is maximum, its angle of projection may be 45° or 135°.
Reason : Wheather θ is 45° or 135° value of range remains the same, only the sign changes.

92. **Assertion :** A body of mass 1 kg is making 1 rps in a circle of radius 1 m. Centrifugal force acting on it is $4\pi^2$ N.

Reason : Centrifugal force is given by $F = \frac{mv^2}{r}$

93. **Assertion :** K.E. of a moving body given by as^2 where s is the distance travelled in a circular path a refers to variable acceleration.

Reason : Acceleration varies with direction only in this case of circular motion.

94. **Assertion :** Centripetal and centrifugal forces cancel each other.

Reason : This is because they are always equal and opposite.

CRITICALTHINKING TYPE QUESTIONS

95. For which angle between two equal vectors $\vec{A}$ and $\vec{B}$ will the magnitude of the sum of two vectors be equal to the magnitude of each vector?
(a) $\theta = 60°$ (b) $\theta = 120°$
(c) $\theta = 0°$ (d) $\theta = 90°$

96. $\vec{A}$ can be written in terms of components as $\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}$. When will $|\vec{A}|$ be zero
(a) $Ax = Ay = 0$ & $Az \neq 0$ (b) $Ax = Ay = Az \neq 0$
(c) $Ax = Ay = Az = 0$ (d) $|\vec{A}|$ can never be zero.

97. Two vectors *A* and *B* lie in a plane, a third vector *C* lies outside this plane, the sum of these vectors *A* + *B* + *C*
(a) can be zero
(b) can never be zero
(c) lies in a plane containing $\vec{A} + \vec{B}$
(d) lies in a plane containing $\vec{A} \times \vec{B}$

98. ABCDEF is a regular hexagon. The centre of hexagon is a point O. Then the value of
$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$ is
(a) $2\overrightarrow{AO}$ (b) $4\overrightarrow{AO}$ (c) $6\overrightarrow{AO}$ (d) Zero

99. For two vectors **A** and **B**, $|\mathbf{A} + \mathbf{B}| = |\mathbf{A} - \mathbf{B}|$ is always true when
(a) $|\mathbf{A}| = |\mathbf{B}| \neq 0$ (b) $\mathbf{A} \perp \mathbf{B}$
(c) $|\mathbf{A}| = |\mathbf{B}| \neq 0$ and A and B are parallel or anti parallel
(d) None of these

100. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$, then the value of α is

(a) 1/2 (b) –1/2
(c) 1 (d) –1

101. The sum of magnitudes of two forces acting at a point is 16 N and their resultant $8\sqrt{3}$ N is at 90° with the force of smaller magnitude. The two forces (in N) are

(a) 11, 5 (b) 9, 7
(c) 6, 10 (d) 2, 14

102. The coordinates of a particle moving in x–y plane at any instant of time t are $x = 4t^2$; $y = 3t^2$. The speed of the particle at that instant is

(a) 10 t (b) 5 t
(c) 3 t (d) 2 t

103. If $\vec{r}$ is the position vector of a particle at time t, r′ is the position vector of the particle at time t′, and $\overrightarrow{\Delta r}$ is the displacement vector, then instantaneous velocity is given by

(a) $V = \lim_{\Delta t \to 0} \frac{\Delta r'}{\Delta t}$ (b) $V = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$

(c) $V = \lim_{\Delta t \to 0} \frac{\Delta r' - \Delta r}{\Delta t}$ (d) $V = \frac{\Delta r}{\Delta t}$

104. For motion in two or three dimensions, the angle between velocity and acceleration is

(a) 0°
(b) 90°
(c) 180°
(d) Any angle between 0° & 180°

105. A particle crossing the origin of co-ordinates at time t = 0, moves in the xy-plane with a constant acceleration a in the y-direction. If its equation of motion is $y = bx^2$ (b is a constant), its velocity component in the x-direction is

(a) $\sqrt{\frac{2b}{a}}$ (b) $\sqrt{\frac{a}{2b}}$

(c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

106. The position of particle is given by $\vec{r} = 2t^2\hat{i} + 3t\hat{j} + 4\hat{k}$, where t is in second and the coefficients have proper units for $\vec{r}$ to be in metre. The $\vec{a}(t)$ of the particle at t = 1 s is

(a) 4 m s^{-2} along y-direction
(b) 3 m s^{-2} along x-direction
(c) 4 m s^{-2} along x-direction
(d) 2 m s^{-2} along z-direction

107. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of the particle is

(a) directed towards the origin
(b) directed away from the origin
(c) parallel to the position vector
(d) perpendicular to the position vector

108. A body of 3kg. moves in X-Y plane under the action of force given by $6t\hat{i} + 4t\hat{j}$. Assuming that the body is at rest at time t = 0, the velocity of body at t = 3 sec is

(a) $9\hat{i} + 6\hat{j}$ (b) $18\hat{i} + 6\hat{j}$

(c) $18\hat{i} + 12\hat{j}$ (d) $12\hat{i} + 68\hat{j}$

109. The coordinates of a moving particle at any time t are given by $x = a\,t^2$ and $y = b\,t^2$. The speed of the particle is

(a) $2t(a+b)$ (b) $2t\sqrt{(a^2+b^2)}$

(c) $2t\sqrt{(a^2-b^2)}$ (d) $\sqrt{(a^2+b^2)}$

110. A boat which has a speed of 5 km h^{-1} in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water is

(a) 1 km h^{-1} (b) 3 km h^{-1}
(c) 4 km h^{-1} (d) $\sqrt{41}$ km h^{-1}

111. If rain falls vertically with a velocity V_r and wind blows With a velocity v_w from east to west, then a person standing on the roadside should hold the umbrella in the direction

(a) $\tan\theta = \frac{V_w}{V_r}$ (b) $\tan\theta = \frac{V_r}{V_w}$

(c) $\tan\theta = \frac{V_{rw}}{\sqrt{Vr^2 + Vw^2}}$ (d) $\tan\theta = \frac{V_r}{\sqrt{Vr^2 + Vw^2}}$

112. If Vr is the velocity of rain falling vertically and Vm is the velocity of a man walking on a level road, and θ is the angle with vertical at which he should hold the umbrella to protect himself, than the relative velocity of rain w.r.t. the man is given by:

(a) $Vrm = \sqrt{Vr^2 + Vm^2 + 2VrVm\cos\theta}$

(b) $Vrm = \sqrt{Vr^2 + Vm^2 - 2VrVm\cos\theta}$

(c) $Vrm = \sqrt{Vr^2 + Vm^2}$

(d) $Vrm = \sqrt{Vr^2 - Vm^2}$

113. A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant the bullet leaves the barrel of the gun, the monkey drops. Pick the correct statement regarding the situation.

(a) The bullet will never hit the monkey
(b) The bullet will always hit the monkey
(c) The bullet may or may not hit the monkey
(d) Can't be predicted

114. A particle moves in a plane with a constant acceleration in a direction different from the initial velocity. The path of the particle is a/an

(a) straight line (b) arc of a circle
(c) parabola (d) ellipse

115. A stone is just released from the window of a moving train moving along a horizontal straight track. The stone will hit the ground following a

(a) straight line path (b) circular path
(c) parabolic path (d) hyperbolic path

116. Two bullets are fired horizontally with different velocities from the same height. Which will reach the ground first?

(a) Slower one
(b) Faster one
(c) Both will reach simultaneously
(d) It cannot be predicted

117. A stone is projected with an initial velocity at an angle to the horizontal. A small piece separates from the stone before the stone reaches its maximum height. Then this piece will

(a) fall to the ground vertically
(b) fly side by side with the parent stone along a parabolic path
(c) fly horizontally initially and will trace a different parabolic path
(d) lag behind the parent stone, increasing the distance from it

118. A ball is thrown from rear end of the compartment of train to the front end which is moving at a constant horizontal velocity. An observer A sitting in the compartment and another observer B standing on the ground draw the trajectory. They will have

(a) equal horizontal and equal vertical ranges
(b) equal vertical ranges but different horizontal ranges
(c) different vertical ranges but equal horizontal ranges
(d) different vertical and different horizontal ranges

119. Two balls are projected simultaneously in the same vertical plane from the same point with velocities v_1 and v_2 with angle θ_1 and θ_2 respectively with the horizontal. If $v_1 \cos \theta_1 = v_2 \cos \theta_2$, the path of one ball as seen from the position of other ball is :

(a) parabola
(b) horizontal straight line
(c) vertical straight line
(d) straight line making 45° with the vertical

120. Two stones are projected from the same point with same speed making angles $45° + \theta$ and $45° - \theta$ with the horizontal respectively. If $\theta \leq 45°$, then the horizontal ranges of the two stones are in the ratio of

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

121. A missile is fired for maximum range with an initial velocity of 20 m/s. If g = 10 m/s^2, the range of the missile is

(a) 40 m (b) 50 m
(c) 60 m (d) 20 m

122. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ ?

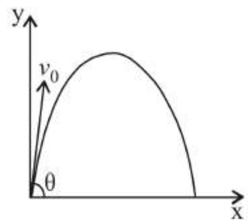
(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45°

123. A projectilel can have the same range R for two angles of projection. If t_1 and t_2 be the times of flight in two cases, then what is the product of two times of flight?

(a) $t_1t_2 \propto R$ (b) $t_1t_2 \propto R^2$
(c) $t_1t_2 \propto 1/R$ (d) $t_1t_2 \propto 1/R^2$

124. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.



(a) $-mg\, v_0 t^2 \cos\theta\, \hat{j}$ (b) $mg\, v_0 t \cos\theta \hat{k}$
(c) $-\frac{1}{2} mg v_0 t^2 \cos\theta \hat{k}$ (d) $\frac{1}{2} mg v_0 t^2 \cos\theta \hat{i}$

125. A particle of mass m is projected with a velocity υ making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

(a) $\frac{\sqrt{3}}{2}\frac{m\upsilon^2}{g}$ (b) zero
(c) $\frac{m\upsilon^3}{\sqrt{2}g}$ (d) $\frac{\sqrt{3}}{16}\frac{m\upsilon^3}{g}$

126. Two projectiles A and B thrown with speeds in the ratio $1 : \sqrt{2}$ acquired the same heights. If A is thrown at an angle of 45° with the horizontal, the angle of projection of B will be

(a) 0° (b) 60°
(c) 30° (d) 45°

127. A body projected at an angle with the horizontal has a range 300 m. If the time of flight is 6 s, then the horizontal component of velocity is

(a) 30 m s^{-1} (b) 50 m s^{-1}
(c) 40 m s^{-1} (d) 45 m s^{-1}

128. A particle of unit mass is projected with velocity u at an inclination α above the horizon in a medium whose resistance is k times the velocity. Its direction will again make an angle α with the horizon after a time

(a) $\frac{1}{k}\log\left\{1-\frac{2ku}{g}\sin\alpha\right\}$ (b) $\frac{1}{k}\log\left\{1+\frac{2ku}{g}\sin\alpha\right\}$

(c) $\frac{1}{k}\log\left\{1+\frac{ku}{g}\sin\alpha\right\}$ (d) $\frac{1}{k}\log\left\{1+\frac{2ku}{3g}\sin\alpha\right\}$

129. The greatest range of a particle, projected with a given velocity on an inclined plane, is x times the greatest vertical altitude above the inclined plane. Find the value of x.
(a) 2 (b) 4 (c) 3 (d) 1/2

130. A body is projected vertically upwards with a velocity u, after time t another body is projected vertically upwards from the same point with a velocity v, where $v < u$. If they meet as soon as possible, then choose the correct option

(a) $t=\frac{u-v+\sqrt{u^2+v^2}}{g}$ (b) $t=\frac{u-v+\sqrt{u^2-v^2}}{g}$

(c) $t=\frac{u+v+\sqrt{u^2-v^2}}{g}$ (d) $t=\frac{u-v+\sqrt{u^2-v^2}}{2g}$

131. A cricket ball is hit at an angle of 30° to the horizontal with a kinetic energy E. Its kinetic energy when it reaches the highest point is

(a) $\frac{E}{2}$ (b) 0

(c) $\frac{2E}{3}$ (d) $\frac{3E}{4}$

132. The range of a projectile is R when the angle of projection is 40°. For the same velocity of projection and range, the other possible angle of projection is
(a) 45° (b) 50°
(c) 60° (d) 40°

133. If the angles of projection of a projectile with same initial velocity exceed or fall short of 45° by equal amounts , then the ratio of horizontal ranges is
(a) 1 : 2 (b) 1 : 3
(c) 1 : 4 (d) 1 : 1

134. A projectile is fired from the surface of the earth with a velocity of 5 ms^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 ms^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in ms^{-2}) given g = 9.8 m/s^2
(a) 3.5 (b) 5.9
(c) 16.3 (d) 110.8

135. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R,θ) on the circle of radius R is (Here θ is measured from the x-axis)

(a) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$

(b) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$

(d) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

136. When a particle is in uniform circular motion it does not have
(a) radial velocity and radial acceleration
(b) radial velocity and tangential acceleration
(c) tangential velocity and radial acceleration
(d) tangential velocity and transverse acceleration

137. A particle moves in a circular orbit under the action of a central attractive force inversely proportional to the distance ‘r’. The speed of the particle is
(a) proportional to r^2 (b) independent of r
(c) proportional to r (d) proportional to 1/r

138. A particle describes uniform circular motion in a circle of radius 2 m, with the angular speed of 2 rad s^{-1}. The magnitude of the change in its velocity in $\frac{\pi}{2}$ s is
(a) 0 m s^{-1} (b) $2\sqrt{2}$m s^{-1}
(c) 8 m s^{-1} (d) 4 m s^{-1}

139. A stone of mass 2 kg is tied to a string of length 0.5 m. If the breaking tension of the string is 900 N, then the maximum angular velocity, the stone can have in uniform circular motion is
(a) 30 rad s^{-1} (b) 20 rad s^{-1}
(c) 10 rad s^{-1} (d) 25 rad s^{-1}

140. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$m with constant tangential acceleration. It the velocity of particle is 80 m/sec at end of second revolution after motion has begun, the tangential acceleration is
(a) 40 π m/sec^2 (b) 40 m/sec^2
(c) 640 π m/sec^2 (d) 160 π m/sec^2

141. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone?
(a) π^2 m s^{-2} and direction along the radius towards the centre.
(b) π^2 m s^{-2} and direction along the radius away from the centre.
(c) π^2 m s^{-2} and direction along the tangent to the circle.
(d) $\pi^2/4$ m s^{-2} and direction along the radius towards the centre.

HINTS AND SOLUTIONS

FACT/DEFINITION TYPE QUESTIONS

1. **(d)** A scalar quantity has only magnitude and the same value for observers with different orientations of axes.

2. **(c)** A vector quantity is defined as the quantity which has magnitude and direction and for which all the mathematical operations are possible only through vector laws of algebra.

3. **(a)** Resultant vector of two vectors $\vec{A}$ & $\vec{B}$ inclined at an angle θ, is given by

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ $\quad \therefore$ if $\theta = 0°$; $\cos 0° = 1$

$\therefore R = \sqrt{A^2 + B^2 + 2AB} = \sqrt{(A+B)^2}$

$R = A + B$

This is the maximum resultant possible.

4. **(c)** The resultant of any three vectors will be cancel out by Fourth vector

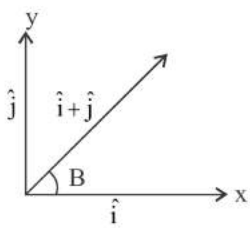
5. **(d)** All the three unit vectors have the magnitude as unity

$\therefore \hat{i} = \hat{j} = \hat{k} = 1$

6. **(c)** $|\hat{i} + \hat{j}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$|\hat{i}| = 1$

$\cos\beta = \frac{1}{\sqrt{2}}$ $\quad \therefore \beta = 45°$



7. **(b)**

8. **(c)**

9. **(c)** The resultant of $\vec{A} \times 0$ is a vector of zero magnitude. The product of a vector with a scalar gives a vector.

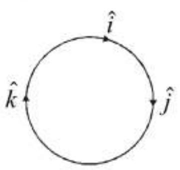
10. **(a)** In a clockwise system,

$\hat{i} \times \hat{j} = \hat{k}$, $\quad \hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

And $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} . \hat{i} = \hat{j} . \hat{j} = \hat{k} . \hat{k} = 1$

$\hat{i} . \hat{j} = \hat{j} . \hat{k} = \hat{k} . \hat{i} = 0$



Therefore, the right option is $\hat{j} \times \hat{k} = \hat{i}$

11. **(b)** 12. **(b)**

13. **(d)** The equation of motion for projectile is $x = x_0 + U_x t + \frac{1}{2} a_x t^2$

$\therefore$ The shape of the trajectory depends on the initial position, initial velocity and acceleration.

14. **(d)** Position vector, $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

Velocity vector

$\vec{v} = \frac{d\vec{r}}{dt}$ $(-a\omega\sin\omega t)\ \hat{i} + (a\omega\cos\omega t)\ \hat{j}$

$(-a\omega\sin\omega t)(a\cos\omega t) + (-a\omega\cos\omega t) + (a\cos\omega t) = 0$

$\Rightarrow \vec{v} \perp \vec{r}$

15. **(c)** $V_y = u\sin\theta - gt_m = 0$

$\therefore t_m = \frac{u_y \sin\theta}{g}$ (time to reach the maximum height)

Total time of flight $T_f = \frac{2(u\sin\theta)}{g}$

$\therefore T_f = 2t_m$

16. **(d)** Horizontal range $= \frac{u^2 \sin 2\theta}{g}$

For maximum range θ = 45°

$\therefore R_{max} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g}$ $\quad (\because \sin 90° = 1)$

17. **(c)** Force due to viscosity, air – resistance are all dissipative forces. Thus in the presence of air – resistance the horizontal component of velocity will decrease, thus for horizontal component of velocity to remain constant, there should be no air-resistance.

18. **(b)** If air resistance is ignored, then there is no acceleration in horizontal direction in projectile motion. Hence the particle move with constant velocity in horizontal direction.

19. **(c)** $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$= 80 - 65 = 15$ km/h

20. **(b)** In both the cases, the initial velocity in the vertical downward direction is zero. So they will hit the ground simultaneously.

21. **(c)** The nature of path is determined by acceleration of particle. For example in uniform circular motion the transverse acceleration is zero & only radial acceleration acts. If a_R (radial acceleration) is zero, then particle go in the direction in which transverse acceleration acts (if it is not zero).

22. **(d)** $T = \frac{2u\sin(\theta - \alpha)}{g\cos\alpha}$

23. **(d)** Velocity and kinetic energy is minimum at the highest point.

$K.E = \frac{1}{2} m v^2 \cos^2\theta$

24. **(b)** Only horizontal component of velocity (u cos θ)

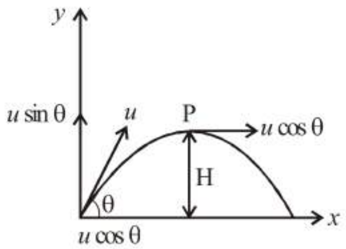
25. **(b)** $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$

$\therefore \quad \tan\theta = 4.$

26. **(c)** $H = \frac{u^2 \sin 45^\circ}{2g} = \frac{u^2}{4g}$

$R = \frac{u^2}{g} = = 4\,H.$

27. **(d)** At maximum height (H) i.e. at point P the vertical component of the projectile $u \sin\theta = 0$ whereas its horizontal component i.e. $u\cos\theta$ remains the same.



28. **(c)** In our discussion, we shall assume that the air resistance has negligible effect on the motion of the projectile.

29. **(c)** If the angle of projection is $\frac{\pi}{4}$, then range $= \frac{v_0^2}{g} \sin(\pi/2)$

$\Rightarrow \quad (R)_{max} = \frac{v_0^2}{g} \qquad [\because \{\sin(\pi/2)\}_{max} = 1]$

30. **(d)** At the highest point of trajectory, the acceleration is equal to g.

31. **(d)** Centripetal acceleration, $a_c = \frac{v^2}{R}$

Where v is the speed of an object and R is the radius of the circle. It is always directed towards the centre of the circle. Since v and R are constants for a given uniform circular motions, therefore the magnitude of centripetal acceleration is also constant. However, the direction of centripetal acceleration changes continuously. Therefore, a centripetal acceleration is not a constant vector.

32. **(a)** 33. **(a)**

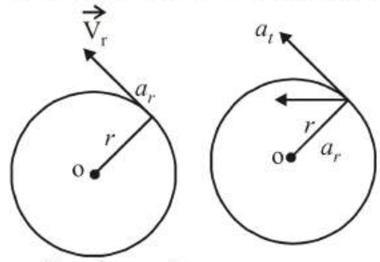
34. **(b)** $\frac{v_1}{v_2} = \frac{r_1\omega}{r_2\omega} = \frac{1}{2}\,[v = r\omega]$

35. **(d)** $\vec{v} = \vec{\omega} \times \vec{r}$

As linear velocity vector $\vec{v}$ is along the tangent to the circular path and angular velocity vector $\vec{\omega}$ is perpendicular to $\vec{v}$, so $\vec{\omega}$ is along the axis of rotation.

36. **(c)** When a particle moves on a circular path with a constant speed, then its motion is said to be a uniform circular motion in a plane. This motion has radial acceleration whose magnitude remains constant but whose direction changes continuously, So $a_r \neq 0$ and $a_t = 0$.

If the circular motion of the particle is not uniform but accelerated then along with the radial acceleration it will have tangential acceleration also and both these acceleration will be mutually perpendicular.

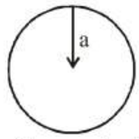


So, $a_r \neq 0$ and $a_t = 0$.

When, $a_r = 0$ and $a_t = 0$ motion is accelerated translatory.

Also, when $a_r = 0$ and then motion is uniform translatory.

37. **(c)** In circular motion with constant speed, acceleration is always inward, its magnitude is constant but direction changes, hence acceleration changes, so does velocity. K.E. is constant.



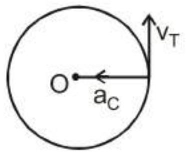
38. **(a)** Body moves with constant speed, it means that it performs uniform circular motion. In this motion the direction of motion is always perpendicular to centripetal force. Hence the work done by centripetal force is always zero

$(dW = \vec{F}_c.\vec{d}r = F_c\,dr\cos\theta = 0, \Rightarrow \theta = 90^\circ)$

39. **(c)** Body moves with constant speed it means that tangential acceleration $a_T = 0$ & only centripetal acceleration a_C exists whose direction is always towards the centre or inward (along the radius of the circle).

40. **(c)** Since the circular motion is uniform, therfore there is no change of angular velocity. Thus angular acceleration is zero.

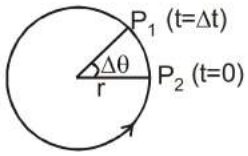
41. **(c)** In uniform circular motion, the body move with v_T (tangential velocity) & a_C. If $a_C = 0$ then it implies that the body is no longer bound to rotate in circle & so no change in the direction of velocity. Hence it move tangentially to the circle outward with velocity v_T.



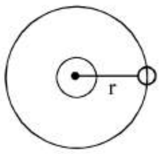
42. **(c)** Since velocity is defined as $v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$

Where $s = r.\theta$, is an arc to circle, which is swept by the particle in Δt time, r is radius of circle which is constant in uniform circular motion & $\Delta\theta$ is angular displacement in Δt time.

Hence if particle in a circle describe equal angles in equal intervals of time, its speed (magnitude of velocity vector) remains same but the direction changes due to centripetal acceleration a_C.



43. **(a)** In circular motion of a particle with constant speed, particle repeats its motion after a regular interval of time but does not oscillate about a fixed point. So, motion of particle is periodic but not simple harmonic.



44. **(a)** In uniform circular motion speed is constant. So, no tangential acceleration.

It has only radial acceleration $a_R = \frac{v^2}{R}$ [directed towards center]

and its velocity is always in tangential direction. So these two are perpendicular to each other.

45. **(d)**

STATEMENT TYPE QUESTIONS

46. **(d)** Addition and subtraction of scalars make sense only for quantities with same units, however multiplication and division of scalars of different unit is possible.

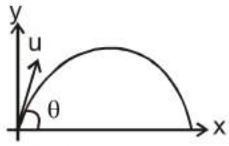
47. **(d)** **48.** **(d)** **49.** **(c)** **50.** **(d)**

51. **(b)** When a body moves on a curved path with a constant speed, it experiences the centripetal acceleration which along the radius. Since velocity acts along the tangent therefore acceleration is perpendicular to the direction of velocity and hence motion.

52. **(d)** While going up, the vertical component of velocity keeps on decreasing due to gravity & thus at highest point it becomes zero. The horizontal component remains constant so at the highest point, if is non-zero. Acceleration due to gravity acts at the heighest point. So it acts vertically downwards.

53. **(c)** In projectile motion, the horizontal range is independent of the mass and depends on the angle of projection according to the relation: $R = \frac{u^2 \sin 2\theta}{g}$

54. **(a)** If we neglect air resistance, horizontal component of velocity is always same.



To find vertical component use equation,

$v_f^2 = v_i^2 - 2g \times h$, $\quad v_i = u\sin\theta$, $h = 0$,

$v_f^2 = u^2 \sin^2\theta - 0$, $v_f = u\sin\theta$

hence $\vec{v} = u\cos\theta\hat{i} - u\sin\theta\hat{j}$

$\therefore$ Speed is same, so K.E. is same.

55. **(c)**

56. **(d)** Centripetal acceleration has a constant magnitude and is always directed towards the centre.

MATCHING TYPE QUESTIONS

57. **(a)** A $\to$ (1) ; B $\to$ (4) ; C $\to$ (2) ; D $\to$ (3)

(A) $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} + 5\hat{j}$ $\therefore$ $\vec{A} + \vec{B} = 6\hat{i} + 8\hat{j}$

(B) $|\vec{A} + \vec{B}| = \sqrt{6^2 + 8^2} = 10$

(C) $\vec{A} - \vec{B} = (2\hat{i} + 3\hat{j}) - (4\hat{i} + 5\hat{j}) = -2\hat{i} - 2\hat{j}$

(D) $|\vec{A} - \vec{B}| = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2}$.

58. **(b)** A $\to$ (2) ; B $\to$ (2) ; C $\to$ (4) ; D $\to$ (3)

(A) $\vec{A} = 3\hat{i} + 4\hat{j}$, $\quad \therefore \quad A = \sqrt{3^2 + 4^2} = 5$

and $\vec{B} = \hat{i} - 2\hat{j}$, $\quad \therefore \quad B = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

(B) $\hat{A} = \frac{\vec{A}}{A} = \frac{3\hat{i} + 4\hat{j}}{5} = 0.6\hat{i} + 0.8\hat{j}$

(C) $\vec{A} + \vec{B} = (3\hat{i} + 4\hat{j}) + (\hat{i} - 2\hat{j}) = 4\hat{i} + 2\hat{j}$

$\therefore \quad |\vec{A} + \vec{B}| = \sqrt{4^2 + 2^2} = \sqrt{20}$

(D) $\vec{A} - \vec{B} = (3\hat{i} + 4\hat{j}) - (\hat{i} - 2\hat{j}) = 2\hat{i} + 6\hat{j}$.

59. **(d)** A $\to$ (1) ; B $\to$ (2) ; C $\to$ (4) ; D $\to$ (3)

(A) $(\vec{A} + \vec{B})/2 = \frac{(\hat{i} + \hat{j}) + (\hat{i} - \hat{j})}{2} = \hat{i}$

(B) $(\vec{A} - \vec{B})/2 = \frac{(\hat{i} + \hat{j}) - (\hat{i} - \hat{j})}{2} = \hat{j}$

(C) $(\vec{A}.\vec{B})/2 = \frac{(\hat{i} + \hat{j}).(\hat{i} - \hat{j})}{2} = \frac{1-1}{2} = 0$

(D) $\frac{(\vec{A} \times \vec{B})}{2} = \frac{(\hat{i} + \hat{j}) \times (\hat{i} - \hat{j})}{2} = \frac{0 - \hat{k} - \hat{k} + 0}{2} = -\hat{k}$

60. **(a)** (A)$\to$(4); (B)$\to$(1); C$\to$(2); (D)$\to$(3)

61. **(b)** (A)$\to$(2); (B)$\to$(3); C$\to$(1); (D)$\to$(4)

62. **(c)** (A)→(2); (B)→(3); C→(1); (D)→(4)

$$U_x = \frac{dx}{dt} = 1$$

and $$U_y = \frac{dx}{dt} = 1 - 2t$$

$$\therefore U_{t=0} = \sqrt{u_x^2 + y_y^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ m/s.}$$

$$a_x = \frac{d^2x}{dt^2} = 0$$

$$d_y = \frac{d^2y}{dt^2} = -2$$

For time of flight,

$$y = 0$$

or $$0 = t - t^2$$

$$\therefore \quad t = 1s.$$

For maximum height,

$$t = \frac{1}{2} s.$$

$$\therefore \quad H = t - t^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4} \text{ m.}$$

63. **(a)** (A)→(1,2); (B)→(1); C→(2); (D)→(4)

Range of the ball in absence of the wall

$$= \frac{u^2 \sin 2\theta}{g} = \frac{20^2 \sin 150°}{10} m = 20m$$

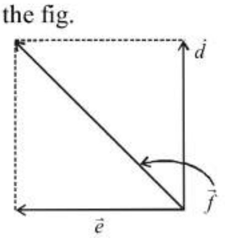
When d < 20m, ball will hit the wall, when d = 25m, ball will fall 5m short of the wall.

When d < 20m, ball will hit the ground, at a distance, x = 20m – d in front of the wall.

DIAGRAM TYPE QUESTIONS

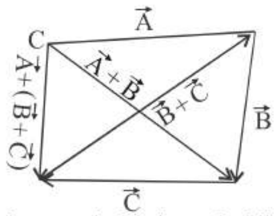
64. **(c)** By the triangle law of vector addition $\vec{C} + \vec{A} = \vec{B}$

65. **(c)** Using the law of vector addition, $(\vec{d} + \vec{e})$ is as shown in the fig.



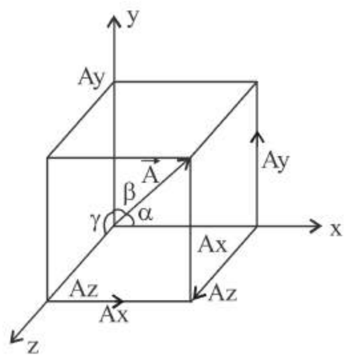
$\therefore \quad \vec{d} + \vec{e} = \vec{f}$

66. **(a)**

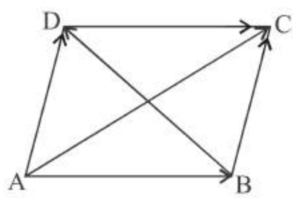


It illustrates the associative law of addition.

67. **(a)**



68. **(a)**



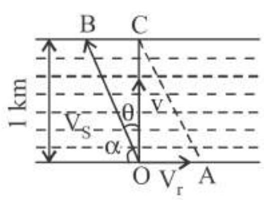
$$\overrightarrow{AC} + \overrightarrow{BD} = (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{BC} + \overrightarrow{CD})$$

$$= \overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$$

$$= \overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB}$$

$$= 2\overrightarrow{BC}$$

69. **(d)** The swimmer will cross straight if the resultant velocity of river flow and swimmer acts perpendicular to the direction of river flow. It will be so if the swimmer moves making an angle α with the upstream. i.e. goes along OB.



70. **(b)** Two bodies will collide at the highest point if both cover the same vertical height in the same time.

$$\text{So } \frac{V_1^2 \sin^2 30°}{2g} = \frac{V_2^2}{2g} \Rightarrow \frac{V_2}{V_1} = \sin 30° = \frac{1}{2}$$

$$\therefore \quad V_2 = \frac{1}{2} V_1$$

71. **(b)** At point B the direction of velocity component of the projectile along Y - axis reverses.

Hence, $\vec{V}_B = 2\hat{i} - 3\hat{j}$

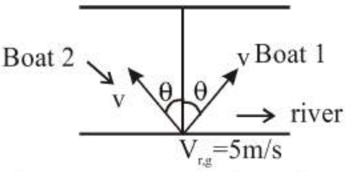
72. **(c)** $a = \frac{v^2}{r} = 1$ cm/s. Centripetal acceleration is directed towards the centre. Its magnitude = 1. Unit vector at the mid point on the path between P and Q is $-(\hat{x} + \hat{y})/\sqrt{2}$.

ASSERTION- REASON TYPE QUESTIONS

73. **(d)** If a vector quantity has zero magnitude then it is called a null vector. That quantity may have some direction even if its magnitude is zero.

74. **(b)** **75.** **(c)** **76.** **(b)** **77.** **(d)** **78.** **(c)**

79. **(b)**



If component of velocities of boat relative to river is same normal to river flow (as shown in figure) both boats reach other bank simultaneously.

80. **(b)** $h = ut - \frac{1}{2}gt^2$ and $v^2 = u^2 - 2gh$;

These equations are independent of mass.

81. **(b)** When a body is projected up making an angle θ the velocity component along-axis remains constant.

$\therefore$ Momentum along x-axis is constant.

Along horizontal, mass and velocity both are constant.

82. **(a)** $R = \frac{u^2 \sin 2\theta}{g}$ If $\theta = 45° + \alpha$

then $R_1 = \frac{u^2 \sin 2(45° + \alpha)}{g} = \frac{u^2 \sin(90° + \alpha)}{g} = \frac{u^2 \cos\alpha}{g}$

If $\theta = 45° - \alpha$

then $R_2 = \frac{u^2 \sin^2(45° - \alpha)}{g} = \frac{u^2 \sin(90° - \alpha)}{g}$

$= \frac{u^2 \cos\alpha}{g}$ $\therefore R_1 = R_2$

83. **(c)** If gravitational force is zero, then $a_y = 0$.

So, $x = u \cos\theta\, t$ and $y = u \sin\theta\, t$

$\therefore$ $y = x \tan\theta$. It represent straight line.

The resultant path of the body depends on initial velocities and acceleration.

84. **(a)** For maximum height $\theta = 90°$, or body must be projected straight upwards. Then

$0 = u^2 - 2gh,$

$\therefore$ $h = \frac{u^2}{2g}$.

85. **(d)** $T = \frac{2u\sin\theta}{g}$, it will maximum, when $\theta = 0°$.

$R_{max} = \frac{u^2}{g}$, for $\theta = 45°$.

86. **(d)** At the highest point of the trajectory,

$v_y = 0,$ and

so, $\vec{P}_y = 0.$

For the two pieces, it is

$\vec{P}_{1y} + \vec{P}_{2y} = 0.$

87. **(d)** Linear momentum during parabolic path changes continuously.

88. **(d)** Statement-1 is false because angles of projection θ and $(90° - \theta)$ give same range but time of flight will be different. Statement-2 is true because in horizontal direction acceleration is zero.

89. **(c)** Maximum horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore$ R_{max}

$= \frac{u^2}{g}$ when $\theta = 45°$

$\therefore$ $R_{max} \propto u^2$

Height $H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow H_{max} = \frac{u^2}{2g}$ when $\theta = 90°$

It is clear that $H_{max} = \frac{R_{max}}{2}$

90. **(d)** $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$

91. **(a)** Range, $R = \frac{u^2 \sin 2\theta}{g}$

when $\theta = 45°$, $R_{max} = \frac{u^2}{g} \sin 90° = \frac{u^2}{g}$

when $\theta = 135°$, $R_{max} = \frac{u^2}{g} \sin 270° = \frac{-u^2}{g}$

Negative sign shows opposite direction.

92. **(a)** From relation

$F = \frac{mv^2}{r} = \frac{m(r\omega)^2}{r} = mr\omega^2$ $[\because v = r\omega]$

$= mr\,(2\pi\nu)^2 = 4\pi^2 mr\nu^2$

Here, $m = 1$kg, $\nu = 1$ *rps*, $r = 1m$

$\therefore$ $F = 4\pi^2 \times 1 \times 1 \times 1^2 = 4\pi^2$ N

93. **(c)** **94.** **(d)**

CRITICALTHINKING TYPE QUESTIONS

95. (b) $|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos 120^\circ}$ $(\theta = 120^\circ)$

$$= \sqrt{A^2+B^2+\cancel{2}AB\left(\frac{-1}{\cancel{2}}\right)} \left(\cos 120^\circ = -\frac{1}{2}\right)$$

$$= \sqrt{A^2+B^2-A(A)}$$

$$= \sqrt{B^2} = B \qquad (\because A = B)$$

96. (c) $\vec{A} = Ax\hat{i}+Ay\hat{j}+Az\hat{k}$

$$|\vec{A}| = \sqrt{Ax^2+Ay^2+Az^2}$$

$\therefore$ Even if one component is non – zero the sum $Ax^2 + Ay^2 + Az^2$ can't be zero.

$\therefore$ for $|\vec{A}| = 0, Ax = Ay = Az = 0.$

97. (b) Given $\vec{A}$ and $\vec{B}$ lie in a plane and vector $\vec{C}$ lies outside this plane.

Resultant vector of $\vec{A}$ and $\vec{B}$ lies in the same plane as that vectors $\vec{A}$ and $\vec{B}$.

Resultant vector of $\vec{A}$, $\vec{B}$ and $\vec{C}$ in non. coplanar vector therefore, their resultant can never be zero.

98. (c) **99. (b)**

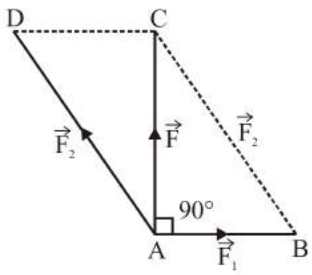
100. For two vectors to be perpendicular to each other

$$\vec{A}\cdot\vec{B} = 0$$

$$(2\hat{i}+3\hat{j}+8\hat{k})\cdot(4\hat{j}-4\hat{i}+\alpha\hat{k}) = 0$$

$$-8 + 12 + 8\alpha = 0 \text{ or } \alpha = -\frac{4}{8} = -\frac{1}{2}$$

101. (d)



In ΔABC,

$$F_2^2 = F^2 + F_1^2 \text{ or } F^2 = F_2^2 - F_1^2$$

$$(8\sqrt{3})^2 = F_2^2 - F_1^2$$

$$192 = (F_2 + F_1)(F_2 - F_1)$$

$$F_2 - F_1 = \frac{192}{16} = 12N \; [\because F_1 + F_2 = 16N]$$

On solving we get,

$F_1 = 2N, F_2 = 14$ N

102. (a) According to the question, at any instant t, $x = 4t^2, y = 3t^2$

$$\therefore v_x = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$$

and $v_y = \dfrac{dy}{dt} = \dfrac{d}{dt}(3t^2) = 6t$

The speed of the particle at instant t.

$$v = \sqrt{v_x^2+v_y^2} = \sqrt{(8t)^2+(6t)^2} = 10t$$

103. (b) Average velocity $= \dfrac{\text{displacement vector}}{\text{time interval}} = \dfrac{\Delta r}{\Delta t}$

Instantaneous velocity is limiting value of average velocity as the time interval approaches zero.

$$\therefore \vec{V} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

104. (d) Along same straight line, velocity & acceleration can be in the same direction, opposite to each other or perpendicular as in circular motion with uniform speed. Thus θ can be anywhere between 0 & 180°.

105. (b) $y = bx^2$

Differentiating w.r.t to t an both sides, we get

$$\frac{dy}{dx} = b2x\frac{dx}{dt}$$

$v_y = 2bxv_x$

Again differentiating w.r.t to t on both sides we get

$$\frac{dv_y}{dt} = 2bv_x\frac{dx}{dt} + 2bx\frac{dv_x}{dt} = 2bv_x^2 + 0$$

$\left[\dfrac{dv_x}{dt}\right.$ = 0, because the particle has constant acceleration along y-direction]

Now, $\dfrac{dv_y}{dt} = a = 2bv_x^2$;

$$v_x^2 = \frac{a}{2b}$$

$$v_x = \sqrt{\frac{a}{2b}}$$

106. (c) $\vec{r} = 2t^2\hat{i}+3t\hat{j}+4\hat{k}$

$$\therefore \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} = (2t^2\hat{i}+3t\hat{j}+4\hat{k}) = 4t\hat{i}+3\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(4t\hat{i}+3\hat{j}) = 4\hat{i}$$

$\therefore$ $\vec{a} = 4\text{ms}^{-2}$ along x-direction

107. (d) $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$

$$\vec{v} = \frac{\vec{d}(r)}{dt} = \frac{d}{dt}\{(a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}\}$$

$$= (-a\omega\sin\omega t)\hat{i} + (a\omega\cos\omega t)\hat{j}$$

$$= \omega[(-a\sin\omega t)\hat{i} + (a\cos\omega t)\hat{j}]$$

$\vec{r}.\vec{v} = 0$

$\therefore$ velocity is perpendicular to the displacement.

108. (a) $F = 6t\hat{i} + 4t\hat{j}$ or $a_x = \frac{6t}{3}, a_y = \frac{4t}{3}$

so $u_x = \int_0^t a_x dt = t^2 \Rightarrow (u_x)_{t=3} = 9$m/sec

and $u_y = \int_0^t a_y dt = \frac{2t^2}{3} \Rightarrow (u_y)_{t=3} = 6$m / sec

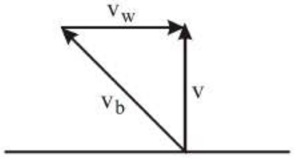
(because u_x & $u_y = 0$ at t = 0 sec)

109. (b) $r = \mathbf{i}\, a\, t^2 + \mathbf{j}\, b\, t^2$, $v = \frac{dr}{dt} = i2at + j2bt$

$\therefore$ Magnitude of $v = \sqrt{(4a^2t^2 + 4b^2t^2)}$

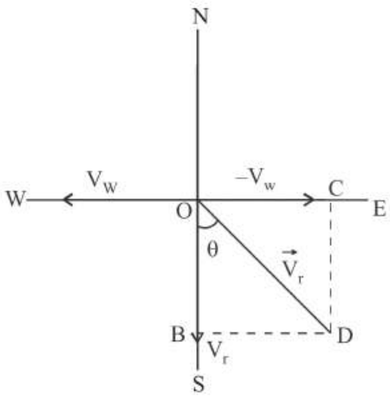
$= 2t\sqrt{(a^2 + b^2)}$

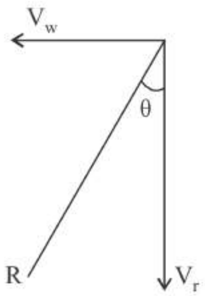
110. (b) $v = \frac{1\,km}{\frac{1}{4}h} = 4\,km\,h^{-1}$, $v_b = 5\,km\,h^{-1}$



$v_w = \sqrt{v_b^2 - v^2} = \sqrt{25-16} = \sqrt{9} = 3\,km\,h^{-1}$

111. (a) Man should hold the umbrella in the direction of the relative velocity of the rain. If Vr $\to$ velocity of rain, $V_W \to$ velocity of wind and $V_{rw} \to$ relative velocity of rain w.r.t. wind

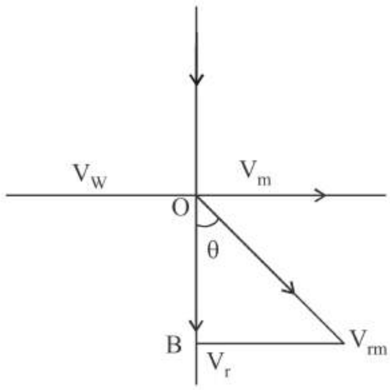




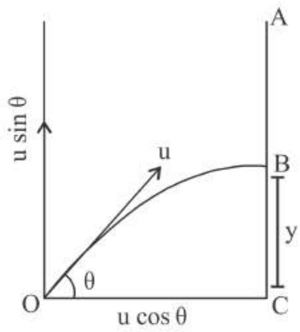
$\therefore \quad \tan\theta = \frac{\vec{V}_w}{\vec{V}_r}$

112. (c) According to pythagorus theorem

$Vrm = \sqrt{Vr^2 + Vm^2}$



113. (b)



$t = \frac{OC}{u\cos\theta} = \frac{x}{u\cos\theta}$

AC = x tan θ

BC = distance travelled by bullet in time t, vertically.

$y = u\sin\theta\, t - \frac{1}{2}gt^2$

$AB = x\tan\theta - (u\sin\theta t - \frac{1}{2}gt^2)$

$= x\tan\theta - (u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2}gt^2)$

$\Rightarrow$ distance trevelled by monkey

$= x\tan\theta - x\tan\theta + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

($\therefore$ bullet will always hit the monkey)

114. (c) Only in case of parabolic motion, the direction and magnitude of the velocity changes, acceleration remains same. Morever, in case of uniform circular motion, the direction changes.

115. (c) The horizontal velocity of the stone will be the same as that of the train. In this way, the horizontal motion will be uniform. The vertical motion will be controlled by the force of gravity. Hence it is accelerated motion. The resultant motion is a parabolic trajectory.

116. (c) The time taken to reach the ground depends on the height from which the projectile is fired horizontally. Here height is same for both the bullets and hence they will reach the ground simultaneously.

117. (b) The piece will fly side by side because the velocity of the piece is the same.

118. (b)

119. (c)

120. (a) Note that the given angles of projection add upto 90°. So, the ratio of horizontal ranges is 1 : 1.

121. (a) For maximum range, the angle of projection, $\theta = 45°$

$$\therefore R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \sin(2\times 45°)}{10} = \frac{400\times 1}{10} = 40\text{m}.$$

122. (c) Yes, the person can catch the ball when horizontal velocity is equal to the horizontal component of ball's velocity, the motion of ball will be only in vertical direction w.r.t person

for that $\frac{v_0}{2} = v_0 \cos\theta$ or $\theta = 60°$

123. (a) $t_1 = \frac{2u\sin\theta}{g}$ and

$$t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore\ t_1 t_2 = \frac{4u^2\cos\theta\sin\theta}{g^2} = \frac{2}{g}\left[\frac{u^2\sin 2\theta}{g}\right] = \frac{2}{g}R,$$

where R is the range.

Hence $t_1 t_2 \propto R$

124. (c) $\vec{L} = m(\vec{r}\times\vec{v})$

$$\vec{L} = m\left[v_0\cos\theta t\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}\right]$$

$$\times\left[v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j}\right]$$

$$= mv_0\cos\theta t\left[-\frac{1}{2}gt\right]\hat{k} = -\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$$

125. (d) Angular momentum of the projectile

$$L = mv_h r_\perp = m(v\cos\theta)\,h$$

where h is the maximum height

$$= m(\upsilon\cos\theta)\left(\frac{\upsilon^2\sin^2\theta}{2g}\right)$$

$$L = \frac{m\upsilon^3\sin^2\theta\cos\theta}{2g} = \frac{\sqrt{3}\,m\upsilon^3}{16g}$$

126. (c) For projectile A

Maximum height, $H_A = \frac{u_A^2\sin^2 45°}{2g}$

For projectile B

Maximum height, $H_B = \frac{u_B^2\sin^2\theta}{2g}$

As we know, $H_A = H_B$

$$\frac{u_A^2\sin^2 45°}{2g} = \frac{u_B^2\sin^2\theta}{2g}$$

$$\frac{\sin^2\theta}{\sin^2 45°} = \frac{u_A^2}{u_B^2}$$

$$\sin^2\theta = \left(\frac{u_A}{u_B}\right)^2\sin^2 45°$$

$$\sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{4}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

127. (b) As we know, $R = u\cos\theta \times t$
Given, $R = 300$ m, $t = 6$ s

$$\therefore\ u\cos\theta = \frac{R}{t} = \frac{300}{6} = 50\text{ms}^{-1}$$

128. (b) Resistance $= kv$ $\left(= k\frac{ds}{dt}\right)$

Equations of motion are

$$\frac{d^2x}{dt^2} = -k\frac{dx}{dt} \quad \text{............. (1)}$$

$$\frac{d^2y}{dt^2} = -k\frac{dy}{dt} - g \quad \text{............. (2)}$$

Integrating (1) and (2) and using the initial conditions, we get

$$\frac{dx}{dt} = u\cos\alpha.e^{-kt} \quad \text{............. (3)}$$

and $k\frac{dy}{dt} + g = (ku\sin\alpha + g).e^{-kt}$

i.e., $\frac{dy}{dt} = \frac{1}{k}[(ku\sin\alpha + g).e^{-kt} - g]$ .... (4)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{[(ku\sin\alpha + g).e^{-kt} - g]}{ku\cos\alpha.e^{-kt}} \quad \text{.... (5)}$$

Direction of projection was α with the horizontal, when the direction of motion again makes the angle α with the horizontal, it really makes the angle $(\pi - \alpha)$ with the horizontal in the sense of the direction of projection. If this happens after the time t, we have from (5),

$$\tan(\pi-\alpha) = \frac{(ku\sin\alpha + g).e^{-kt} - g}{ku\cos\alpha.e^{-kt}}$$

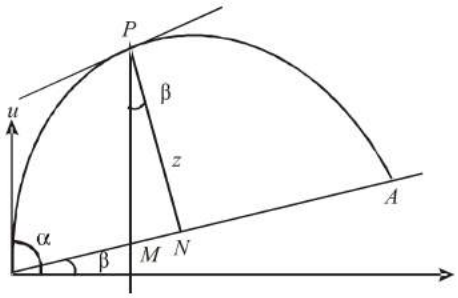
i.e., $$-\tan\alpha = \frac{(ku\sin\alpha + g) - ge^{-kt}}{ku\cos\alpha}$$

i.e., $$-ku\sin\alpha = ku\sin\alpha + g - g.e^{-kt}$$

or $$e^{kt} = 1 + \frac{2ku}{g}\sin\alpha$$

or $$t = \frac{1}{k}\log\left(1 + \frac{2ku}{g}\sin\alpha\right)$$

129. (b) P be the point where the tangent is parallel to the inclined plane. If $PN = z$ be perpendicular from P on the inclined plane and PM the vertical altitude of P then evidently for all points on the path, P is the point where z is the greatest and consequently PM is greatest.



Now for the point P, velocity perpendicular to the inclined plane is zero. Now the velocity and acceleration perp. to the plane at O is u sin $(\alpha - \beta)$ and $g\cos\beta$ and this velocity becomes zero at P.

$\therefore \quad 0 = u^2\sin^2(\alpha-\beta) - 2g\cos\beta.z$

$$z = \frac{u^2\sin^2(\alpha-\beta)}{2g\cos\beta}$$

For max. range $\alpha = \frac{\pi}{4} + \frac{\beta}{2}$ or $\alpha - \beta = \frac{\pi}{4} - \frac{\beta}{2}$

Hence, $$z = \frac{u^2}{2g\cos\beta}\sin^2\left(\frac{\pi}{4} - \frac{\beta}{2}\right)$$

$$= \frac{u^2}{4g\cos\beta}\left[1 - \cos\left(\frac{\pi}{2} - \beta\right)\right]$$

$$= \frac{u^2}{4g\cos\beta}(1-\sin\beta)$$ or $PM = z\sec\beta$

$$= \frac{u^2}{4g\cos^2\beta}(1-\sin\beta) = \frac{u^2}{4g(1+\sin\beta)} = \frac{1}{4}$$ (max. range)

$\Rightarrow$ Maximum range = 4 × PM

130. (b) Let the two bodies meet each other at a height h after time T of the projection of second body. Then before meeting, the first body was in motion for time $(t + T)$ whereas the second body was in motion for time T.

The distance moved by the first body in time $(t + T)$

$= u(t+T) - \frac{1}{2}g(t+T)^2$.

And the distance moved by the second body in time T

$= vT - \frac{1}{2}gT^2 = h$ (supposed above). .... (1)

$\because$ The two bodies meet each other,

$\therefore$ They are equidistant from the point of projection.

Hence, $u(t+T) - \frac{1}{2}g(t+T)^2 = vT - \frac{1}{2}gT^2$

or $u(t+T) - \frac{1}{2}g(t^2 + 2tT) = vT$

or $gt^2 + 2t(gT - u) + 2(v-u)T = 0$ .... (2)

Also from (1) we get, $h = vT - \frac{1}{2}gT^2$

$\therefore \frac{dh}{dT} = v - gT$

$\therefore$ h increases as T increases

$\therefore$ T is minimum when h is minimum i.e., when

$\frac{dh}{dT} = 0$, i.e. when $v - gT = 0$ or $T = v/g$.

Substituting this value of T in (2), we get

$gt^2 + 2t(v-u) + 2(v-u)(v/g) = 0$

or $g^2t^2 - 2gt(u-v) + 2v(u-v) = 0$

or $$t = \frac{2g(u-v) + \sqrt{4g^2(u-v)^2 + 8vg^2(u-v)}}{2g^2}$$

or $$t = \frac{u - v + \sqrt{u^2 - v^2}}{g}$$

neglecting the negative sign which gives negative value of t.

131. (d) Kinetic energy at the highest point is

$$E_{top} = \frac{1}{2}mu^2\cos^2\theta$$

Here $\frac{1}{2}mu^2 = E$

and $\cos\theta = \frac{\sqrt{3}}{2}$ $[\because \theta = 30°]$

$\therefore E_{top} = \frac{3}{4} \times E$

132. (b) Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

Range is same for angle of projection θ and (90° – θ)

133. (d) For complementary angles of projection (45° + α) and (45° – α) with same initial velocity u, range R is same.

$\theta_1 + \theta_2 = (45° + \alpha) + (45° - \alpha) = 90°$

134. (a) Horizontal range = $\frac{u^2 \sin 2\theta}{g}$ so $g \propto u^2$

or $\frac{g_{planet}}{g_{earth}} = \frac{(u_{planet})^2}{(u_{earth})^2}$

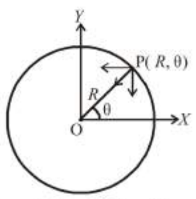
Therefore $g_{planet} = \left(\frac{3}{5}\right)^2 (9.8\ m/s^2)$

$= 3.5\ m/s^2$

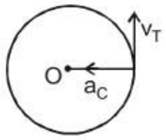
135. (c) Clearly

$\vec{a} = a_c \cos\theta(-\hat{i}) \ + a_c \sin\theta(-\hat{j})$

$= \frac{-v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$



136. (c) It has only tangential velocity v_T & radial acceleration or centripetal acceleration a_C.



137. (b) Centripetal force, $F = \frac{mv^2}{r}$, so, $F \propto \frac{1}{r}$ so v is independent of r.

138. (c) Given, $\omega = 2$ rad s^{-1}, r = 2 m, $t = \frac{\pi}{2}$ s

Angular displacement, $\theta = \omega t = 2 \times \frac{\pi}{2} = \pi$ rad

Linear velocity, $v = r \times \omega = 2 \times 2 = 4$ m s^{-1}

∴ change in velocity, $\Delta v = 2v \sin\frac{\theta}{2} = 2 \times 4 \times \sin\left(\frac{\pi}{2}\right)$

$= 8$ m s^{-1}

139. (a) As $T = mr\omega^2$

or $\omega^2 = \frac{T}{mr} = \frac{900}{2 \times 0.5} = 900 \Rightarrow \omega = 30$ rad s^{-1}

140. (b) Circumference of circle is $2\pi r = 40m$

Total distance travelled in two revolution is 80m. Initial velocity u = 0, final velociy v = 80 m/sec

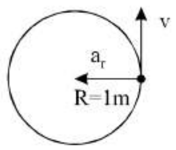
so from

$v^2 = u^2 + 2as$

$\Rightarrow (80)^2 = 0^2 + 2 \times 80 \times a$

$\Rightarrow a = 40 m/sec^2$

141. (a) $a_r = \omega^2 R$



$a_r = (2\pi 2)^2 R = 4\pi^2 2^2 R = 4\pi^2\left(\frac{22}{44}\right)^2 (1)$ $\left[\because v = \frac{22}{44}\right]$

$a_t = \frac{dv}{dt} = 0$

$a_{net} = a_r = \pi^2$ ms^{-2} and direction along the radius towards the centre.